Implicit Differentiation Review

Implicit Differentiation

When we have to take the derivative of a function where y is part of the equation itself, we need to apply implicit differentiation. The trick is to simply add a y' every time we differentiate a function with a y. Then we need to move all the terms contain y' to one side the equation and everything else to the other side. Factor out y' and solve for y'.

Example

Differentiate the following function with respect to x

$$x^2 + y^2 = 9$$

Example

Differentiate the following function with respect to x.

$$2y^2 + 4xy + x^2 = 3$$

Since the function is not explicitly defined for y, we must use implicit differentiation.

$$4yy' + 4xy' + 4y + 2x = 0$$
$$4yy' + 4xy' = -4y - 2x$$
$$y'(4y + 4x) = -4y - 2x$$

$$y' = \frac{-4y - 2x}{4y + 4x}$$

To differentiate 4xy we use the product rule

$u = 4x \quad v = y$

$u' = 4 \quad v' = y'$

$(4xy)' = 4xy' + 4y$

3.5 Question 5

Find $\frac{dy}{dx}$ by implicit differentiation

$$x^2 - 4xy + y^2 = 4$$

3.5 Question 9

Find $\frac{dy}{dx}$ by implicit differentiation

$$\frac{x^2}{x+y} = y^2 + 1$$

3.5 Question 13

Find $\frac{dy}{dx}$ by implicit differentiation

$$\sin(x + y) = \cos x + \cos y$$

3.5 Question 17

Find $\frac{dy}{dx}$ by implicit differentiation

$$2xe^y + ye^x = 3$$

3.5 Question 21

Find $\frac{dy}{dx}$ by implicit differentiation

$$e^{x/y} = x - y$$