MATH 2271
Chapter 11.1

Orthogonal Functions

Inner Product of 2 functions

$(f_1, f_2) = \int_a^b f_1(x) f_2(x) dx$

Orthogonal Functions
When the inner product is equal to 0, the two functions or orthogonal.

$(f_1, f_2) = \int_a^b f_1(x) f_2(x) dx = 0$

Orthogonal Set

A set of real-valued functions $\{\phi_0(x), \phi_1(x), \phi_2(x), \dots\}$ is orthogonal if

$$\left(\phi_m, \phi_n\right) = \int_a^b \phi_m(x)\phi_n(x)dx = 0 \quad m \neq n$$

Norm (Length)

$$\left|\left|\phi_n(x)\right|\right|^2 = \int_a^b \phi_n^2(x)dx$$

$$\left|\left|\phi_n(x)\right|\right| = \sqrt{\int_a^b \phi_n^2(x)dx}$$

Orthonormal Set

If $\left|\left|\phi_n(x)\right|\right| = 1$ for an orthogonal set of functions $\{\phi_n(x)\}$

Example
Show that the given functions are orthogonal on the indicated interval.

$f_1(x) = x, f_2(x) = x^2;\ [-2,2]$

Example
Show that the given functions are orthogonal on the indicated interval.

$f_1(x) = x, f_2(x) = \cos(2x);\ \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Example
Show that the given set of functions is orthogonal on the indicated interval. Find the norm of each function in the set.

$\{\sin x, \sin 3x, \sin 5x, \ldots\}; \quad \left[0, \frac{\pi}{2}\right]$

Example
The functions $f_1(x) = x$ and $f_2(x) = x^2$ are orthogonal on the interval $[-2, 2]$. Find constaints c_1 and c_2 such that $f_3(x) = x + c_1x^2 + c_2x^3$ is orthogonal to both f_1 and f_2 on the same interval.