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### Test 1 Crash Course

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Must Know These Integrals!

$$\int \sin(ax) \, dx = -\frac{1}{a} \cos(ax) + C \qquad \int \cos(ax) \, dx = \frac{1}{a} \sin(ax) + C$$
$$\int \tan(ax) \, dx = \frac{1}{a} \ln|\sec(ax)| + C \qquad \int \sec(ax) \, dx = \frac{1}{a} \ln|\sec(ax) + \tan(ax)| + C$$

$$\int \sec^2 x \, dx = \tan x + C \qquad \qquad \int \csc^2 x \, dx = -\cot x + C$$
$$\int \sec x \tan x \, dx = \sec x + C \qquad \qquad \int \csc x \cot x \, dx = -\csc x + C$$



#### Integrals involving sine and cosine functions $\int \sin^m x \cos^n x \, dx$

Case 1: The power of sine is odd and the power of cosine is even

- a) Save one sine factor and convert the other sine's to cosines using  $sin^2x = 1 cos^2x$
- b) Apply a simple 'u' substitution with u = cosx and solve

Case 2: The power of sine is even and the power of cosine is odd

- a) Save one cosine factor and convert the other cosines to sines using  $\cos^2 x = 1 \sin^2 x$
- b) Apply a simple 'u' substitution with u = sinx and solve

Case 3: The powers on both sine and cosine are even:

Use the half angle identities:  $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$  OR  $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$ You may also have to use  $sinx cosx = \frac{1}{2}sin(2x)$ 

#### Example

 $\int \cos^3 x \sin^4 x \, dx$ 



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#### Example

 $\int \sin^5 x \cos^4 x \, dx$ 



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#### Example

 $\int \frac{1}{\sec^2 x} dx$ 



#### Additional Integrals involving sine and cosine functions:

**Case 1**:  $\int \sin mx \cos nx \, dx$ 

Use the identity  $\sin A \cos B = \frac{1}{2} [\sin(A - B) + \sin(A + B)]$ 

**Case 2**:  $\int \sin mx \sin nx \, dx$ 

Use the identity  $\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$ 

**Case 3**:  $\int \cos mx \cos nx \, dx$ 

Use the identity  $\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$ 

#### Example

 $\int \sin(2x)\cos(3x)dx$ 



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Use the identity  $\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$ 

**Case 3**:  $\int \cos mx \cos nx \, dx$ 

Use the identity  $\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$ 

#### Example

 $\int x\sin(x)\cos(2x)dx$ 



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#### Integrals involving tangent and secant functions $\int tan^m x \sec^n x \, dx$

**Case 1**: The power of tanx is odd and the power of secant x is even

- a) Save one secxtanx and convert all other tanx's to secx using  $tan^2x = sec^2x-1$
- b) Apply a simple 'u' substitution with u = secx and solve

Case 2: The power of tanx is even and the power of secant x is even

- a) Save one  $\sec^2 x$  and convert all other secx's to tanx using  $\sec^2 x = \tan^2 x + 1$
- b) Apply a simple 'u' substitution with u = tanx and solve

Case 3: Apply integration by parts usually, though other methods may also work.

#### Example

 $\int \tan^5 x \sec^6 x \, dx$ 



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#### Integrals involving tangent and secant functions $\int tan^m x \sec^n x \, dx$

**Case 1**: The power of tanx is odd and the power of secant x is even

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Case 2: The power of tanx is even and the power of secant x is even

- a) Save one  $\sec^2 x$  and convert all other secx's to tanx using  $\sec^2 x = \tan^2 x + 1$
- b) Apply a simple 'u' substitution with u = tanx and solve

**Case 3**: Apply integration by parts usually, though other methods may also work.

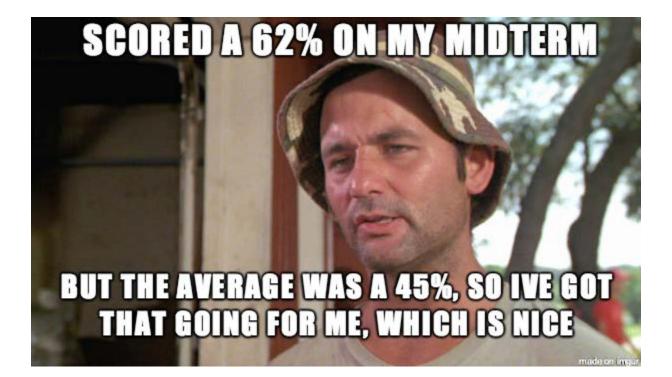
#### Example

 $\int \frac{\tan^2 x}{\cos^4 x} dx$ 



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