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Test 1 Crash Course

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Improper Integrals

Type 1 Improper Integral – Infinite Intervals
In this type of integral, one or both limits is $-\infty/\infty$

A) Single ∞ bound

$$\int_{a}^{\infty} f(x)dx = \lim_{t\to\infty}\int_{a}^{t} f(x)dx$$

$$\int_{-\infty}^{b} f(x)dx = \lim_{t\to-\infty}\int_{t}^{b} f(x)dx$$

If the limit exists, the improper integrals converges.
If the limit does not exist, the improper integrals diverges.

B) Both bounds ∞

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{c} f(x)dx + \int_{c}^{\infty} f(x)dx$$

If both improper integrals on the right converge, then the improper integral on the left converges and it's the sum of the two on the right. When given an improper integral with interval $(-\infty,\infty)$, it **must** be split using a value that lies between $(-\infty,\infty)$**.**

Example

$$\int_{1}^{\infty}\left(\frac{1}{x^2}\right)dx$$

Example

$$\int_{-\infty}^{\infty} e^{3x}dx$$

Example

$$\int_0^{\infty} x^2 e^{-x^3}\,dx$$

Example

$$\int_{-\infty}^{0} xe^{x}dx$$

Type 2 Improper Integral – Discontinuous Integrals

This type of improper integral does not have one of the bounds going to infinity. Rather, the integral is discontinuous at one of the bounds or at some point between the two bounds.

Note: Going forward, always do a quick check to see where the integral may be discontinuous and compare against your bounds to determine if you have an improper integral present.

1. If $f(x)$ is discontinuous at a, then $\int_a^b f(x)dx = \lim_{t\to a^+} \int_t^b f(x)dx$
2. If $f(x)$ is discontinuous at b, then $\int_a^b f(x)dx = \lim_{t\to b^-} \int_a^t f(x)dx$
3. If $f(x)$ is discontinuous at c, where $a < c < b$, then split the integral up into two before evaluating. Then, if both integrals converge, the original does as well.

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Example

$$\int_0^3 \frac{5}{x-3}dx$$

Example

Write a formal definition of the following integral and solve it.

$$\int_0^1 \ln(x)dx$$

Example

$$\int_0^5 \frac{x}{x-2}dx$$

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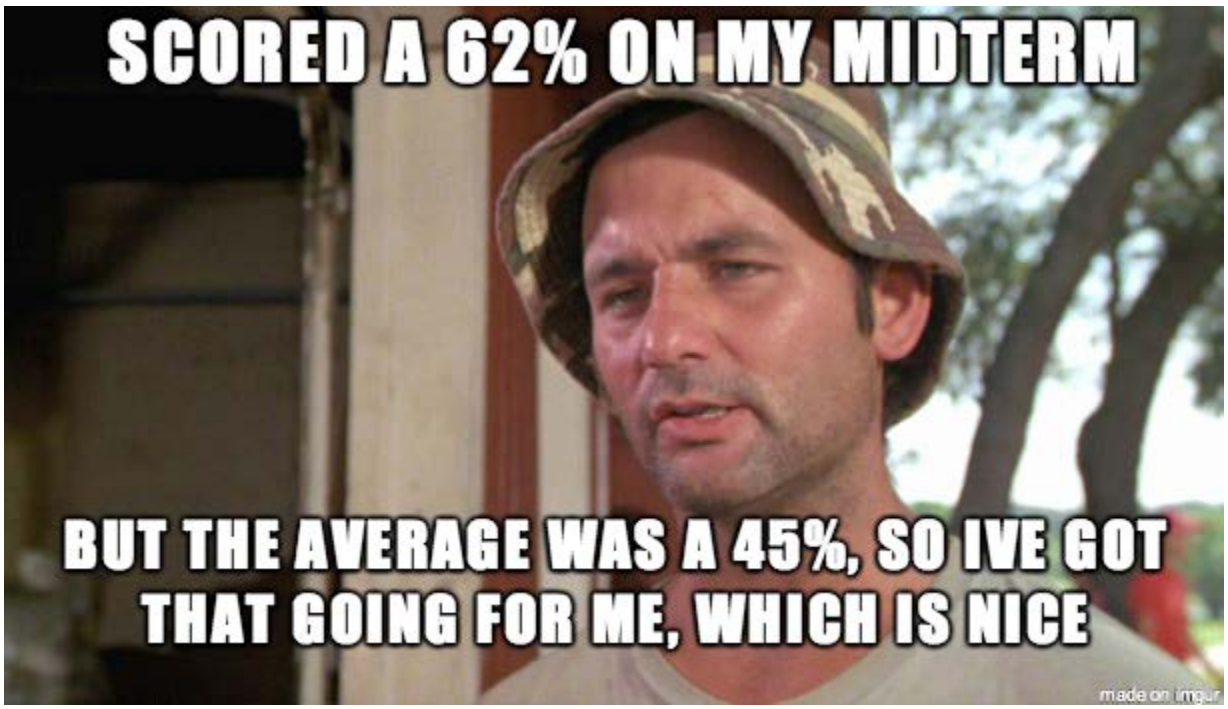

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