MATH1505.com

We Make Math Easy.

Chapter 5.4

Tutorial Length
1 Hr 20 Mins

Homework ~ Tutorials ~ Past Tests

Important

Math 1505 is a HUGE course. Many students fear the course, but you don't need to, you've got us! The keys to success are to practice as many types of problems as possible and not to fall behind. Each chapter builds on the concepts of a previous chapter so it's crucial to understand the material from one chapter before moving on.

This is where MATH1505.COM comes in. We have developed extensive tutorial videos for each section that will give you a quick overview of the theory before we jump in to examples. Our goal is to make things as simple as possible. We will go through MANY examples in order to ensure you understand the concept. We want to show that one concept can be tested in multiple different ways. By making your way through all the questions, you will see different variations and learn new techniques that will make MATH 1505 a breeze. We'll show you shortcuts, easy tricks to remember, and even go through past test questions.

In short, if you're reading this, you're already on the right path. Your success is our success and we wish you the best with this course.

MATH1505.com

Helpful Tips

- Each question has a 4 digit video ID code. If you only want to watch a specific example, just search for the 4 digit code in the playlists.

- Some sections are very long. Consider breaking it down into smaller periods of time (1 hour chunks) in order to efficiently absorb the information.

- When going through tutorial videos, if you are having a particularly difficult time with a question, skip it and come back later. Sometimes the brain just needs a bit of a break!

- Keep all of your MATH1505.com booklets in a binder. This way when it's time to do a final review for a test, you can quickly go through the material. Try to circle or highlight key points. These items will stand out when you begin reviewing.

- If you've purchased access to past tests, don't go through those questions until you feel you've learned all the material. Then go through as many past tests as possible in preparation for your actual test. Once you go through the solutions, you will see where you still have issues and what you still need to review.

Policy Reminder:

While sharing is caring, any user accounts found to be shared between students will be terminated with no refunds. Additionally, access to all premium content will expire after the final exam. If you have a deffered exam, please e-mail within 14 days of the final exam to make arrangements for extended access.

Contact

Questions, Concerns, Comments? info@math1505.com
Please note we are unable to offer tutoring assistance over e-mail.

Substitution

$\int (2x+1)^2\,dx$ $\int \left(\frac{3x^4-2x^2+3}{x^4}\right)dx$ $\int 5x^4 e^{x^5}\,dx$ $\int \frac{\sqrt{lnx+5}}{x}dx$

The method of substitution converts a 'hard' integral into an 'easy' integral. The integral is converted from the current variable to 'U', so sometimes this method is called U-Substitution.

This method is most often used when we see a term and it's derivative in the integral. The derivative can be off by a constant factor however. The method of substitution basically undoes the chain rule of finding a derivative.

Step 1: Select u - (1) u is often the harder of the two terms
(2) u is often the denominator
(3) u is often the exponent of e (or any other exponential base)
(4) u is often in the brackets
(5) We usually do not include the outside exponent when selecting u

Step 2: Find $dx = \frac{du}{u'}$

Step 3: Substitute u and dx back and integrate. **<u>You should only have u's in your integral.</u>** If you have any x's remaining, look at the equation from step 1 and see if you can replace the x variable directly, or by rearranging the equation from step 1.

Step 4: Substitute back for u

Example (VID_9428)

$\int 5x^4 e^{x^5}\,dx$ $\int x^4 e^{x^5} dx$

Substitution Summary

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Example (VID_9623)

$$\int \left(\frac{5x^4}{\sqrt[3]{6x^5 - 8}} \right) dx$$

Integration/Antiderivative Rules

$$\int kf(x)dx = kF(x) + C$$

$$\int [f(x) \pm g(x)]dx = F(x) \pm G(x) + C$$

$$\int kdx = kx + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad x \neq -1$$

$$\int x^{-1} dx = \int \frac{1}{x} dx \ = \ln|x| + C$$

$$\int e^x dx = e^x \ + C$$

$$\int e^{kx} dx = \frac{e^{kx}}{k} + C$$

$$\int a^x dx = \frac{a^x}{ln(a)} + C$$

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Example (VID_5523)

$$\int \frac{3}{xln^4x}dx$$

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Example (VID_8314)

$$\int \frac{6x+4}{3x^2+4x-1} dx$$

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Example (VID_9568)

$$\int \frac{6x+4}{(3x^2+4x-1)^3}dx$$

Substitution Summary

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Step 3 - Substitute, simplify, integrate

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Example (VID_5459)

$$\int \frac{e^{3x}}{e^{3x}-4}dx$$

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Example (VID_5935)

$$\int \frac{\sqrt{lnx+5}}{x}dx$$

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Example (VID_1015)

$$\int 4x\sqrt{x^2 - 3}dx$$

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Example (VID_5140)

$$\int \sqrt{x^2 - 4x}(x-2)dx$$

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Example (VID_1063)

$$\int \frac{x}{x+1} dx$$

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Example (VID_0661)

$$\int \sqrt{2x+1}dx$$

Substitution Summary

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Example (VID_9515)

$$\int 2^x(2^x - 4)dx$$

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Example [VID_

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$$

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Example

$$\int x^2 \sin(x^3)dx$$

Definite Integrals Using Substitution

PDF ID: 1-07-04-003-00-0

If presented with a definite integral that requires substitution to find the antiderivative, there are two possible way to solve the problem.

Method 1) Find the general antiderivative by rewriting it as an indefinite integral & then using substitution. Once the general antiderivative is found, substitute u back in and then plug in the boundaries of integration

Method 2) If you want to avoid substituting u back in at the end, change the boundaries of integration at the beginning so that you can evaluate as soon as the antiderivate is found in terms of u

Example (VID_7907)

$$\int_0^1 4x(x^2+1)^5 dx$$

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Example (VID_4390)

$$\int_0^{\sqrt{5}} x\sqrt{x^2+4}\,dx$$

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Example (VID_4857)

$$\int_0^1 \frac{e^{4t}}{(e^{4t}+2)^3}\,dt$$

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PDF ID: 1-07-04-006-00-0

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Example (VID_2316)

$$\int_1^4 \frac{\ln^2 x}{x}\,dx$$

Definite Integrals Using Substitution PDF ID: 2-05-04-002-00-0

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Example

$$\int_0^{\pi} \cos x \cos(\sin x)\, dx$$